# MOOC Course- Regression Analysis and Forecasting - January 2017

## Assignment 1

[1] Which of the following transformations make the model $y = \frac{x}{\beta_0 x - \beta_1}$ linear in $\beta_0$ and $\beta_1$?

A. $y^* = y^{-2}$, $x^* = x^{-2}$

B. $y^* = y^{-1}$, $x^* = x^{-1}$

C. $y^* = y^2$ , $x^* = x^2$

D. $y^* = y^{-1/2}$ , $x^* = x^{-1/2}$

[2] Which of the statements are correct about the model $y = \beta_0 \exp(\beta_1 x)$?

Statement 1 : Model is nonlinear.

Statement 2 : Model is linear in the parameters $\ln \beta_0$ and $\beta_1$.

Statement 3 : Model can be linearized using the transformed variables $y^* = \ln y$ and $x^* = x$.

Statement 4 : Model is linear in the parameters $\ln \beta_0$ and $\exp(\beta_1)$.

A. Statements 1, 2 and 3 are correct.

B. Statements 1, 3 and 4 are correct.

C. Statements 2 and 3 are correct.

D. All the statements 1, 2, 3 and 4 are correct.

[3] Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \epsilon$ where $\beta_0$ is known. The ordinary least squares estimator of $\beta_1$ based on the observations $(x_i, y_i), i = 1, 2, \ldots, n$ is

A. $\beta_0 \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$.

B. $\frac{\sum_{i=1}^n (y_i - \beta_0) x_i}{\sum_{i=1}^n x_i^2}$.

C. $\frac{\sum_{i=1}^n (y_i - \bar{y}) x_i}{\sum_{i=1}^n x_i^2}$.

D. $\frac{\sum_{i=1}^n (y_i - \bar{y} - \beta_0)(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$.

[4] Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ i = 1, 2 \ldots, n$ where $\epsilon_i$'s are identically and independently distributed with mean 0, variance $\sigma^2$ and do not necessarily follow the normal distribution. Let $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,\ \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$. The covariance between the least squares estimators of $\beta_0$ and $\beta_1$ is

A. $-\frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

B. $-\frac{\bar{x}\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

C. Zero.

D. $\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2\sigma^2}{\bar{x}^2}$.

[5] Consider a simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i,\ E(\epsilon_i) = 0,\ Var(\epsilon_i) = \sigma_i^2, i = 1, 2, \ldots, n$, where $\sigma_i^2, i = 1, 2, \ldots, n$ are assumed to be is known. An estimator of $\beta_1$ based on the minimization of $\sum_{i=1}^{n} \epsilon_i^2$ in this case is

A. $\frac{\sum_{i=1}^{n}\left(\frac{(x_i - \bar{x})(y_i - \bar{y})}{\sigma_i^2}\right)}{\sum_{i=1}^{n}\left(\frac{(x_i - \bar{x})^2}{\sigma_i^2}\right)}$

B. $\frac{\sum_{i=1}^{n}\sigma_i^2(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}\sigma_i^2(x_i - \bar{x})^2}$.

C. $\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$.

D. $\frac{\sum_{i=1}^{n}\left(\frac{(x_i - \bar{x})(y_i - \bar{y})}{\sigma_i}\right)}{\sum_{i=1}^{n}\left(\frac{(x_i - \bar{x})^2}{\sigma_i}\right)}$

**Note: Questions [6] and [7] are based on the following data:**

The weight and systolic blood pressure of 6 randomly selected persons are obtained as follows:

| Observation number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Weight | 165 | 167 | 180 | 155 | 212 | 175 |
| Blood pressure | 130 | 133 | 150 | 128 | 151 | 146 |

Considering the weight to be explanatory variable ($x$) and the blood pressure to be study variable ($y$), the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $\epsilon_i \sim N(0, \sigma^2)$, $i = 1, 2, \ldots, 6$ is fitted.

[6] The ordinary least squares estimates of $\beta_0$, $\beta_1$ and $\sigma^2$ are obtained. Which of the following represents the correct results?

A. $\hat{\beta}_0 = 62.9,\ \hat{\beta}_1 = 0.44,\ \hat{\sigma}^2 = 176$

B. $\hat{\beta}_0 = 26.5,\ \hat{\beta}_1 = 0.15,\ \hat{\sigma}^2 = 176$

C. $\hat{\beta}_0 = 62.9,\ \hat{\beta}_1 = 0.44,\ \hat{\sigma}^2 = 44$

D. $\hat{\beta}_0 = 26.5,\ \hat{\beta}_1 = 0.15,\ \hat{\sigma}^2 = 44$

[7] The standard errors ($se$) of ordinary least squares estimates of $\beta_0$ and $\beta_1$ are obtained. Which of the following represents the correct results?

A. $se(\hat{\beta}_0) = 0.15,\ se(\hat{\beta}_1) = 26.5$

B. $se(\hat{\beta}_0) = 176,\ se(\hat{\beta}_1) = 0.44$

C. $se(\hat{\beta}_0) = 0.44,\ se(\hat{\beta}_1) = 176$

D. $se(\hat{\beta}_0) = 26.5,\ se(\hat{\beta}_1) = 0.15$

[8] Which of the following test statistic is used to test $H_0 : \beta_0 = 0$ in the model $y = \beta_0 + \beta_1 x + \epsilon$, $\epsilon \sim N(0, \sigma^2)$ for a sample of size 60 and $\sigma^2$ is unknown?

A. $Z$-statistic.

B. $t$-statistic.

C. Anyone of $Z$ or $t$-statistic.

D. $\chi^2$-statistic.

[9] Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i, i = 1, 2, \ldots 20$ where $\epsilon_i$'s are identically and independently distributed following $N(0, \sigma^2)$ where $\sigma^2$ is unknown. The following outcome for testing $H_0 : \beta_0 = 6$ is obtained at 5% level of significance. The value of $t$-statistic is 2.78 and $p$-value is 0.08. Which of the following decision is correct?

A. Reject $H_0$.

B. Accept $H_0$.

C. No decision can be concluded.

D. Data is inadequate.

[10] In the simple linear regression model, $y_i = \beta x_i + \epsilon_i$, $\epsilon_i \sim N(0, \sigma^2), i = 1, 2, \ldots, n$, an unbiased estimator of $\sigma^2$ is

A. $\frac{1}{(n-2)\sum_{i=1}^n x_i^2}\left[\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n x_i y_i)^2\right]$

B. $\frac{1}{(n-1)\sum_{i=1}^n x_i^2}\left[\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n x_i y_i)^2\right]$

C. $\frac{1}{n\sum_{i=1}^n x_i^2}\left[\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n x_i y_i)^2\right]$

D. $\frac{1}{(n+1))\sum_{i=1}^n x_i^2}\left[\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n x_i y_i)^2\right]$

## Solution to Assignment 1

Answer of Question 1 – B

Answer of Question 2 – A

Answer of Question 3 – B

Answer of Question 4 – B

Answer of Question 5 – C

Answer of Question 6 – A

Answer of Question 7 – A

Answer of Question 8 – C

Answer of Question 9 – B

Answer of Question 10 – B